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Lessons from a Toy LIDAR Simulation

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Abstract

Backscattered LIDAR data are simulated in a atmosphere including molecular and aerosol components. The aerosol extinction length *versus* height above the LIDAR is extracted following Fernald (*Klett*-like analysis). Variations in the assumed aerosol backward scattering phase function result in large variations in the reconstructed aerosol extinction lengths. In contrast the analysis provides a reliable estimate of the aerosol *scale height*. This fact, coupled with the significant correlation between assumed aerosol phase function and reconstructed extinction length, holds the promise for an aerosol transmission uncertainty correction meeting the Auger 10% uncertainty goal.

1 Introduction

The Auger fluorescence data must be corrected for transmission losses [1] from Rayleigh scattering (on the molecular atmosphere) and from Mie scattering on aerosols in the atmosphere [1]. The aerosol transmission correction depends on the aerosol extinction length which in turn varies with height above the fluorescence detectors. The aerosol extinction length at ground level will be monitored by the horizontal attenuation length (HAM) monitors [1,2] and/or by the backscattered LIDARs [1,3] operating horizontally. The variation of the aerosol extinction length with height above the Auger air fluorescence detectors will be monitored using backscattered LIDARs. In these measurements the LIDAR laser beams are directed at various (non-zero) angles, α, to the horizontal.

This note addresses the analysis of the backscattered LIDAR data for measurements at non-zero values of α. I am aware of at least 3 procedures for analyzing these data:

- use the traditional Klett [4] or Fernald [5] procedure(s),
- use ratios of LIDAR signals from the same height but at different values of α [1,6],
- use multi-parameter fits to the LIDAR signals *versus* time.

The Klett or Fernald technique has been used by both Auger [3] and/or HiRes/TA groups [7]. It is easy to apply but involves one significant assumption. The ratio technique is less easy to apply and requires 1-dimensionality in the atmosphere. The multi-parameter fit is computer intensive and probably relies on knowing the absolute efficiency of the LIDAR to good precision (*e.g.* better than 10%).

As the Klett/Fernald procedure is both popular and simple, the goal of this note is to study the sensitivity of the results to the initial input assumption. This study uses the Fernald analysis. In the Fernald solution the input assumption, or parameter, is the aerosol phase function at 180°; this is a priori unknown.

The concept of this study is to simulate an ideal LIDAR and an (ideal) 1-dimensional atmosphere: where the molecular and aerosol components are both known and easily varied. The LIDAR signals from this simulation are then analyzed following Fernald. The value of the aerosol phase function at 180° assumed in the (Fernald) analysis is varied from $5\times \sim 0.25\times$ the actual value (in the simulation).

2 Model Parameters and Analysis Results

The atmosphere is modeled using the US Standard Atmosphere [8] plus Rayleigh scattering for the molecular component. The aerosols are modeled using 3 parameters:

- aerosol horizontal extinction length, $\Lambda^a(0)$ at the height of the fluorescence detectors, $z \equiv 0$ (nominally 1500m above sea level),
- aerosol mixing height, h_m,
- aerosol scale height, h_s.

Thus the aerosol extinction length *versus* height, $\Lambda^a(z)$, is given by:

$$\Lambda^a(z) = \Lambda^a(0) \qquad (z \leq h_m)$$

and:

$$\Lambda^a(z) = \Lambda^a(0) \cdot e^{(z-h_m)/h_s} \qquad (z > h_m)$$

In this notation the aerosol optical depth, $\tau^a(z)$, is:

$$\tau^a(z) = \int_0^z \frac{dz}{\Lambda^a(z)}$$

which for $z >> h_m + h_s$ gives $\tau^a = (h_m + h_s)/\Lambda^a(0)$. Typical values used in the study are: $\Lambda^a(0) = 20$km, $h_m = 0$m and $h_s = 1200$m. Studies are also done with: $h_m = 500$m, $h_s = 1200$m and with: $h_m = 0$m and h_s varied in the interval: 900m$\leq h_s \leq$ 1500m.

The Fernald analysis requires the LIDAR signal *versus* time, the density of the molecular atmosphere *versus* height and the Rayleigh and assumed Mie/aerosol phase functions for backwards (180°) scattering. The values used in the LIDAR simulations are: $\frac{1}{\sigma^a}(\frac{d\sigma^a}{d\Omega})_{180^\circ} = 0.05$ or 0.025, and $\frac{1}{\sigma^m}(\frac{d\sigma^m}{d\Omega})_{180^\circ} = 3/(8\pi) = 0.1194$. Simulations are also done where the aerosol phase function at 180° is varied with height:

$$\frac{1}{\sigma^a}(\frac{d\sigma^a(z)}{d\Omega})_{180^\circ} = \frac{1}{\sigma^a}(\frac{d\sigma^a(0)}{d\Omega})_{180^\circ} \cdot (1 \pm \frac{z(m)}{7500m})$$

The value of 7500m is arbitrary but matches the maximum height of the LIDAR signals used in this study. The LIDAR is modeled with a 5mJ/pulse, 355nm laser, 0.5m^2 mirror, overall efficiency (photons to P.E.s) of 15% and 10Mhz sampling. These results are all from simulated operation at an angle $\alpha = 30°$ (to the horizontal).

The Fernald analysis (of the simulated LIDAR data) uses the reciprocal of the phase functions at 180° denoted S_1 (aerosols) and $S_2 = (8\pi)/3$ (molecular). The reconstructions use as a central value: $S_1 = 1/0.05 = 20$ or $S_1 = 1/0.025 = 40$ (for the two different LIDAR simulations). The (assumed) values of S_1 are then varied over the range: $0.2 \sim 4$ times the central value(s).

The Fernald analysis steps inward or outward from the starting point. This study starts at an elevation of $\sim$ 7000m and steps inwards from there. The output of the analysis is the aerosol reconstructed extinction length *versus* height, $\Lambda^r(z)$. The values of z (in this toy study) correspond to each digitization of the simulated LIDAR signal; for $\alpha = 30°$ and 100ns sampling the z-binning is 7.5m.

A comparison of the reconstructed aerosol extinction length, $\Lambda^r(z)$, to the (actual) extinction length in the simulation, $\Lambda^a(z)$, is shown if Fig. 1 at a height of 500m (above the LIDAR). The good news is that $\Lambda^r = \Lambda^a$ when $S_1^r = S_1^a$, where S_1^r is the (assumed) value of the aerosol phase function (at 180°) used in the Fernald analysis and S_1^a is the (input) value used in the simulation. The bad news is that an incorrect guess for S_1^r results in an incorrect value for Λ^r. The fact that *e.g.* $\Lambda^r(500m)$ and S_1^r/S_1^a are correlated means that knowledge of $\Lambda^r(500m)$ from horizontal LIDAR measurements (or from the HAMs) will constrain S_1^r to be near the true value (see Fig. 1). A possibly important detail is that the $\Lambda^r(500m)$ - S_1^r/S_1^a correlation weakens as S_1^a increases. Thus in the limit of $S_1^a \to \infty$, *i.e.* the aerosol phase function at $180° \to 0$, there is (appropriately) no correlation. To be conservative this study has used relatively large values of the aerosol scattering phase function at 180°.

While using the $\Lambda^r(z)$ to reconstruct the observed LIDAR signal might help constrain S_1^r, tests with this simulation/analysis find a weaker constraint than possible from Fig. 1. On the other hand it is likely that measurements of the aerosol phase function, from side scattered observations of the LIDAR laser beam [1,9] and/or from the dedicated *aerosol phase function* light source [10], will also help constrain S_1^r/S_1^a, where by S_1^a I now mean the true value at the Auger experiment.

The comparison of $\Lambda^r(z)$ with $\Lambda^a(z)$ at z=500m (above) is somewhat arbitrary. In practice the ratio $\Lambda^r(z)/\Lambda^a(z)$ is relatively insensitive to the value of z and/or to the value of S_1^r/S_1^a. This fact can be used to extract the aerosol scale height from the reconstructed $\Lambda^r(z)$. For heights, z, where $\Lambda^r(z)$ varies exponentially with height, then:

$$h_s(\ (z_2 + z_1)/2\ ) \quad = \quad \frac{(z_2 - z_1)}{ln(\ \Lambda^r(z_2)/\Lambda^r(z_1)\ )}$$

This quantity is evaluated (in steps of 20 digitizations or every $\sim$ 150m vertically) and plotted in Fig. 2 for a variety of different simulations and for a variety of S_1^r/S_1^a. The characteristic decrease of h_s above $\sim$ 4000m reflects the limitations of the Fernald solution as z approaches the starting height ($\sim$ 7000m) of the analysis (as well as the small but non-zero aerosol contribution even at 7000m).

The points with the largest value of h_s in Fig. 2 correspond to a single simulation done with input value h_s = 1500m and analyzed with $S_1^r/S_1^a = 1$. The points with the smallest value of h_s correspond to a single simulation done with input value h_s = 900m (again analyzed with $S_1^r/S_1^a = 1$). These are clearly distinct from all the other simulations run with an input value of h_s = 1200m. The spread of the simulations with input h_s = 1200m comes from variations in S_1^r/S_1^a between 0.2 and 4. In practice the knowledge of the approximate value of $\Lambda^a(500m)$ combined with the correlation in Fig. 1 probably limits $0.7 < S_1^r/S_1^a < 1.5$. Thus the Fernald analysis should provide a good estimate of the aerosol scale height, h_s. This in turn should improve the linking of $\Lambda^r(z)$ with the $z = 0$ values (from horizontal LIDAR and HAM measurements). This linking is straight forward apart from the possible complication of a mixing layer (of height h_m). A mixing layer can be detected if the LIDAR data are available at those heights, *i.e.* for $z \leq h_m$).

From the above it is plausible that the Fernald analysis of the backscattered LIDAR data can measure the aerosol scale height, h_s, and bound the reciprocal aerosol phase function at 180°, S_1^r, to: $0.7 < S_1^r/S_1^a < 1.5$ where we imply that S_1^a is the actual value (near z=0). To see if this is good enough, Fig. 3 shows the fractional aerosol transmission correction error, dT/T, (from our combined simulation plus analysis) for $0.5 < S_1^r/S_1^a < 2.0$ for the case with $S_1^a = 20$. In all the analyses summarized in Fig. 3 $|dT/T| < 2\%$ [11]. However this is without inclusion of a *slant* correction (*i.e.* light from showers is not viewed vertically but rather at a large slant angle from the vertical). If the aerosol transmission correction is to have an uncertainty of less than 10% at angle β, then for typical viewing angles of $\beta = 80°$ from the vertical: $|dT/T| < 0.1 \cdot cos(\beta) \approx 0.017$. This is achieved for approximately the same subset of assumed values $0.7 \leq S_1^r/S_1^a \leq 1.5$ (see Fig. 3) for the aerosol phase function at 180° as should be attainable using the information in Fig. 1 and 2.

3 Summary

Backscattered LIDAR data are simulated in a atmosphere including molecular and aerosol components. The aerosol extinction length *versus* height above the LIDAR is extracted following Fernald (*Klett*-like analysis). Variations in the assumed aerosol backward scattering phase function result in large variations in the reconstructed aerosol extinction lengths. In contrast the analysis provides a reliable estimate of the aerosol *scale height*. This fact, coupled with the significant correlation between assumed aerosol phase function and reconstructed extinction length, holds the promise for an aerosol transmission uncertainty correction meeting the Auger 10% uncertainty goal.

Acknowledgements

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References

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[11] Also included in Fig. 3 are two simulations that included a variation with height in the aerosol phase function (see text). When these simulations were analyzed with $S_1^r/S_1^a = 1$ the variation is found to be less than cases where the simulation has a constant aerosol phase function (at 180°) and the analysis has $S_1^r/S_1^a \leq 0.7$ or $S_1^r/S_1^a \geq 1.5$.

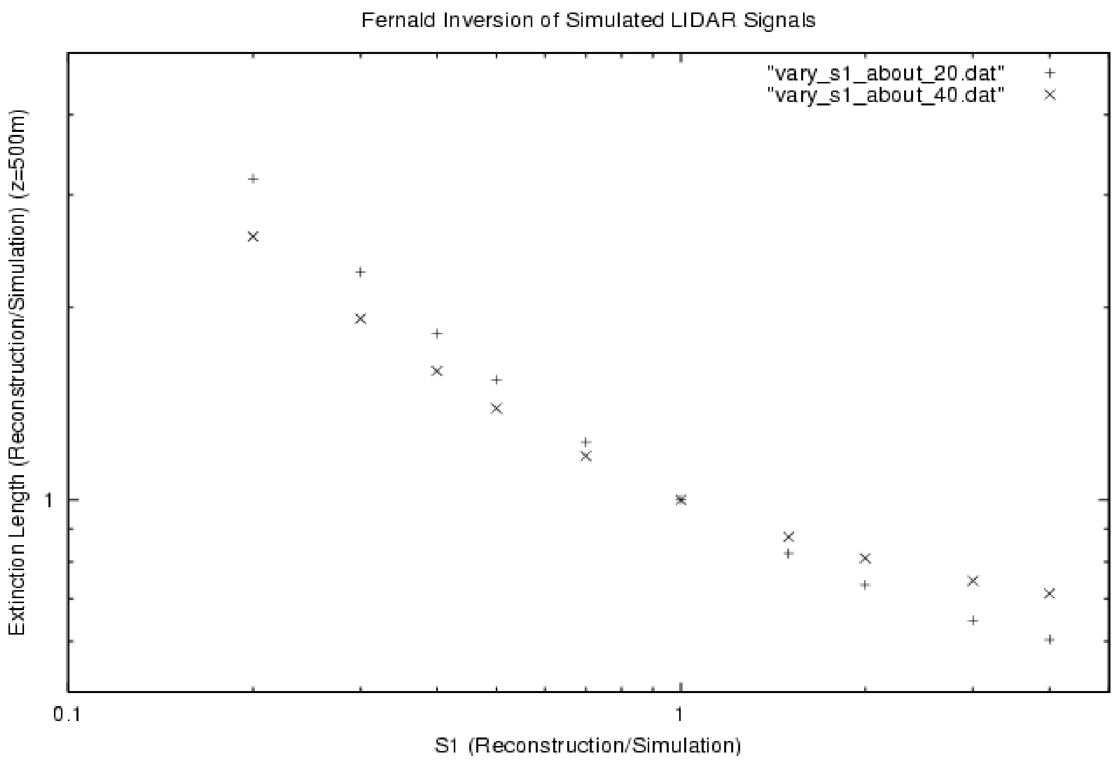

Fig. 1: Plot of $\Lambda^r(500m)/\Lambda^a(500m)$ *versus* S_1^r/S_1^a where S_1^r is the value for the reciprocal aerosol phase function assumed in the Fernald analysis and S_1^a is the (input) value used in the LIDAR simulation.

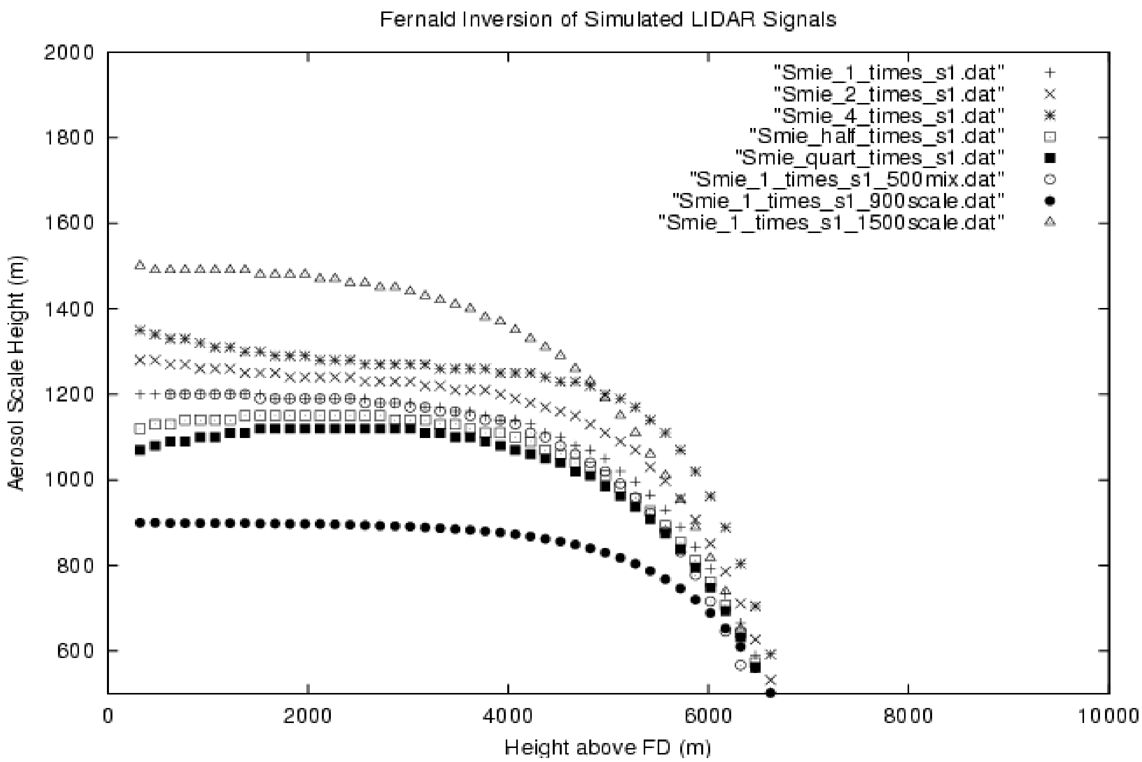

Fig. 2: Plot of reconstructed aerosol scale height, h_s from a variety of different atmospheric simulations and different values for S_1^r/S_1^a used in the Fernald analysis (see text). $S_1^a = 20$ for this study.

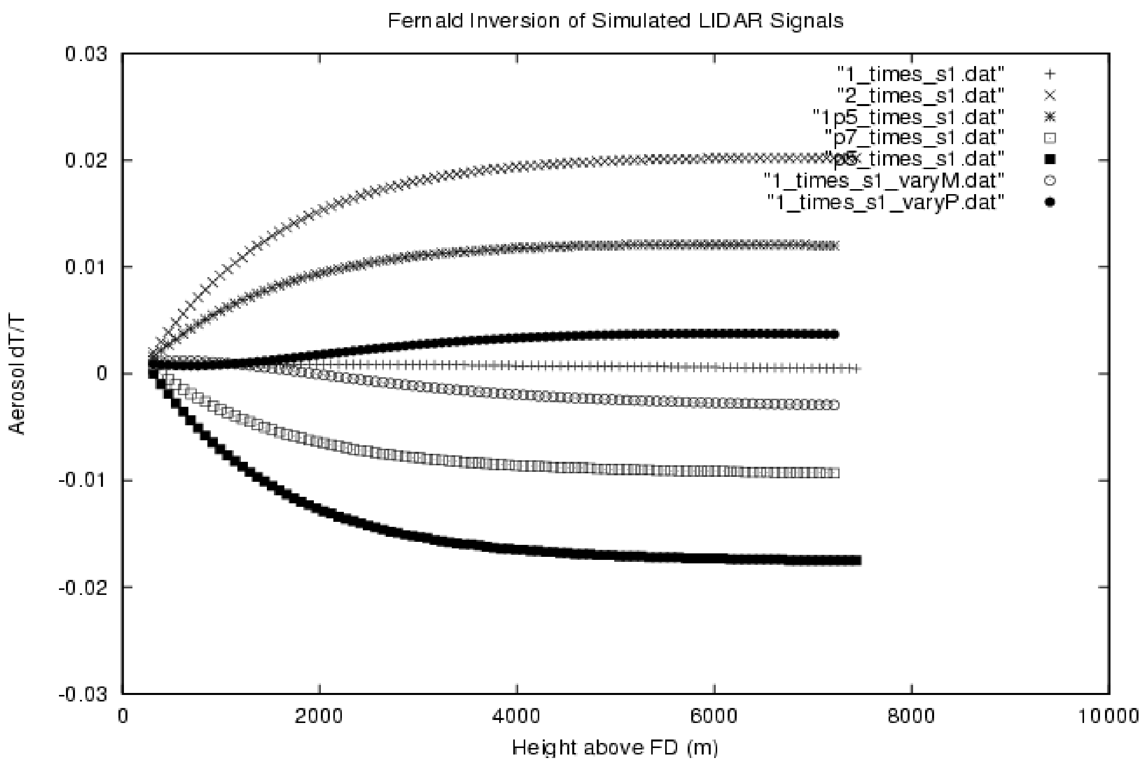

Fig. 3: Plot of fractional error: $dT/T = (1 - T^r/T^a)$, where T^r is the reconstructed aerosol transmission factor and T^a is the actual (input) value, *versus* height, z. Different assumed values for S_1^r/S_1^a (see text) are plotted as different curves. The analysis starts at an arbitrary height, $z \sim 250$m. $S_1^a = 20$ for this study.

Appendix

The body of the text may have left the reader confused as to the actual output of the Fernald inversion. The purpose of this short appendix is to show the output aerosol extinction length, $\Lambda^r(z)$, for a few representative simulations:

1. Fig. A-1 shows the reconstructed aerosol extinction length for three distributions of aerosols: (h_m, h_s) = (0m, 1200m) or (200m, 1000m) or (400m, 800m) with $h_m + h_s$ = 1200m. For this reconstruction the correct aerosol backward phase function was used, *i.e.* $S_1^r/S_1^a = 1.0$. For all cases the aerosol extinction length at $z = 0$m is 20km; thus all of the $\Lambda^r(z)$ should extrapolate to 20,000m (vertical axis in the Fig. A-1). The *gap* between $z = 0$ and $z \approx 250$m in Fig. A-1 results from the simulation starting at 250m above the fluorescence detectors.

2. Fig. A-2 shows one of the simulations, with (h_m, h_s) = (400m, 800m), reconstructed with three different values of S_1^r/S_1^a = 0.7, 1.0, or 1.5. As noted in the body of the text, different values of S_1^r/S_1^a result in reconstructed aerosol extinction lengths, $\Lambda^r(z)$, that differ by a multiplicative factor, see Fig. 1.

3. Fig. A-3 shows a simulation where the aerosol extinction length was varied in steps. Thus for 0m$< z <$ 500m, $\Lambda^a(z) = \Lambda^a(0\text{m}) \cdot e^{(250m)/h_s}$ (a constant). This pattern was continued with 4-total steps in z. Then above $z = 2000$m the aerosol extinction length varied as: $\Lambda^a(z) = \Lambda^a(0\text{m}) \cdot e^{z/h_s}$. The Fernald inversion, which used $S_1^r/S_1^a = 1.0$, reproduced the input structure exactly.

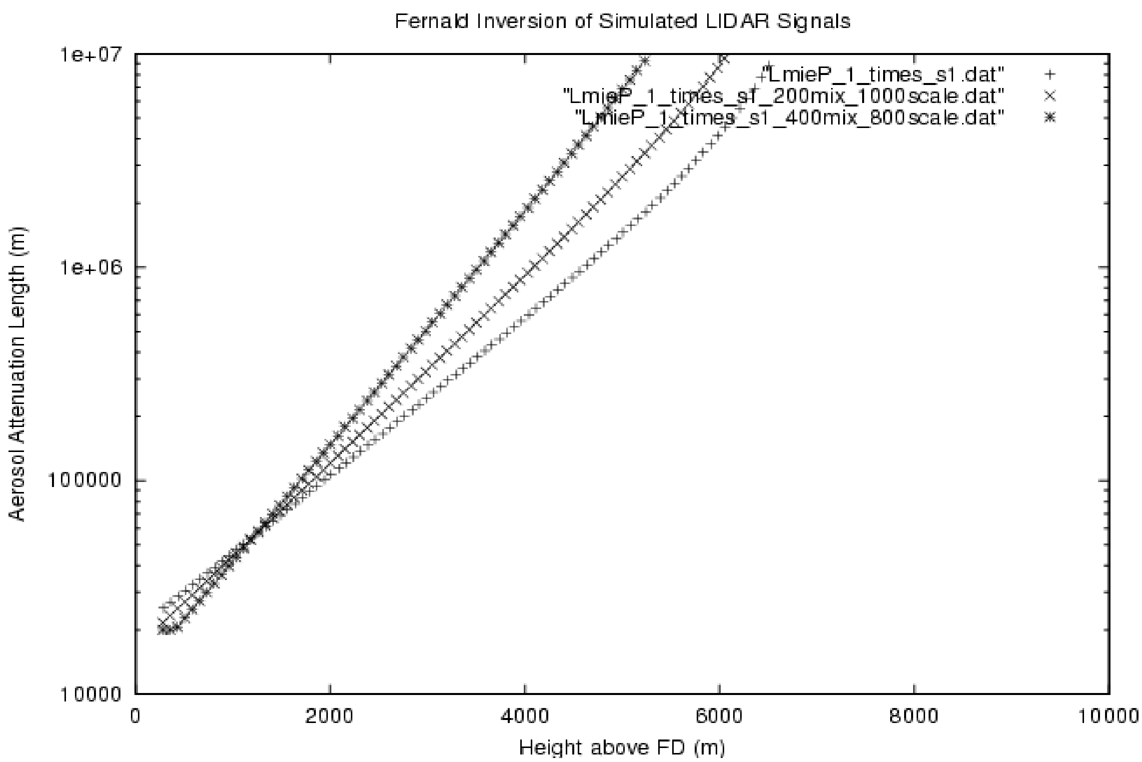

Fig. A-1: Plot of reconstructed aerosol extinction length *versus* height above the detector. See text for details of the input simulation. For the reconstruction $S_1^r/S_1^a = 1.0$.

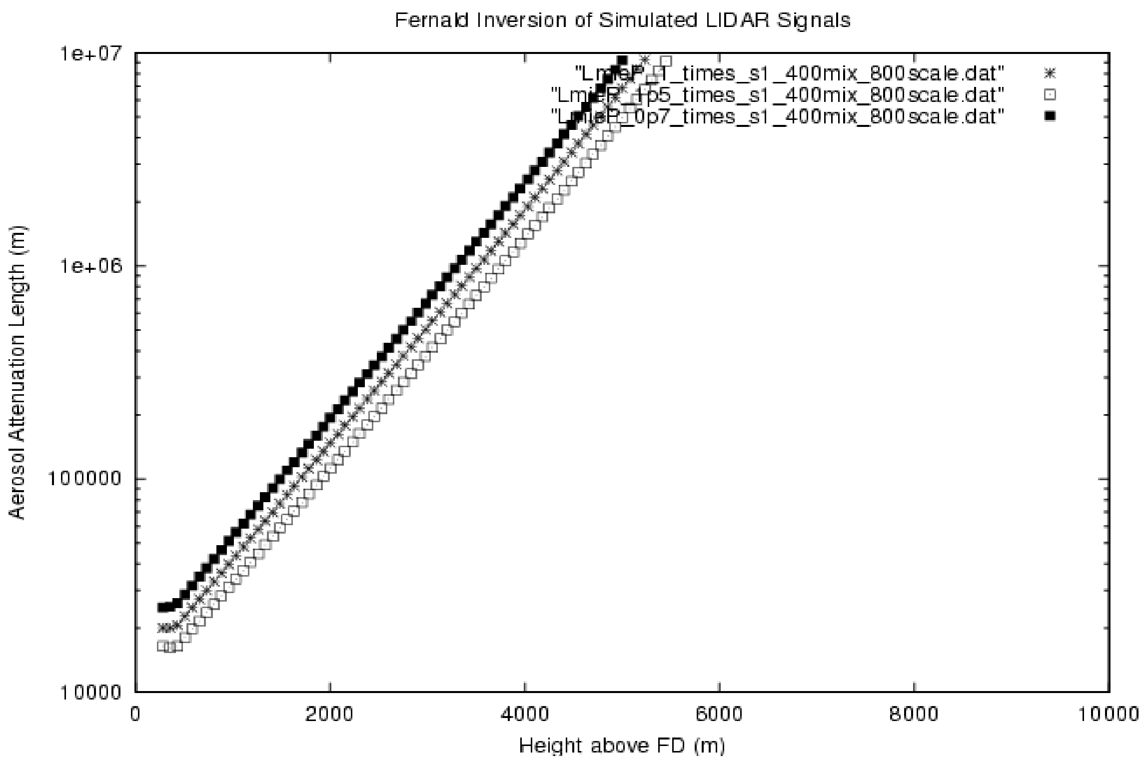

Fig. A-2: Plot of reconstructed aerosol extinction length *versus* height above the detector. See text for details of the input simulation. For the reconstruction $S_1^r/S_1^a = 0.7$ or 1.0 or 1.5.

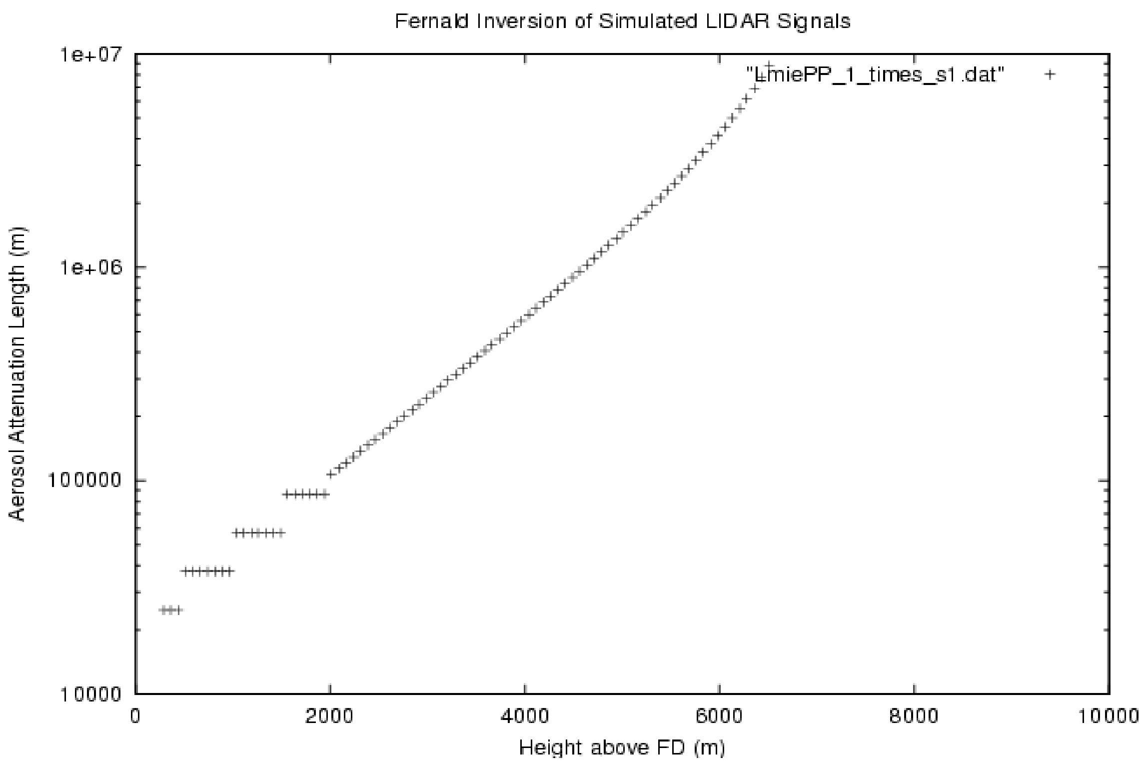

Fig. A-3: Plot of reconstructed aerosol extinction length *versus* height above the detector. See text for details of the input simulation. For the reconstruction $S_1^r/S_1^a = 1.0$.